Henkin measures on strictly pseudoconvex domains

Abstract: We call a complex regular Borel measure μ on the boundary of the open unit ball $\mathbb{B}_d$ of $\mathbb{C}^d$ *Henkin*, if for all sequences (f_n) in the ball algebra $A(\mathbb{B}_d)$ with $\|f_n\|_\infty \leq 1$ for all $n \in \mathbb{N}$ and such that $(f_n(z))$ converges to 0 for all $z \in \mathbb{B}_d$, we have

$$\lim_{n\to\infty} \int_{\partial\mathbb{B}_d} f_n \, d\mu = 0.$$

Henkin measures on $\mathbb{B}_d$ appear in a number of different contexts in the function theory on the unit ball. For instance, a compact subset of $\partial\mathbb{B}_d$ has numerous interpolating properties with respect to $A(\mathbb{B}_d)$ if and only if it is a zero set with respect to every Henkin measure. In recent years, the theory of Henkin measures on $\mathbb{B}_d$ has been generalized to so-called Mult$(\mathcal{H})$-*Henkin* measures. These measures are defined in relation to the multiplier algebras of certain reproducing kernel Hilbert spaces on $\mathbb{B}_d$. This is due to several authors, including Bickel, Clouâtre, Davidson, Hartz and McCarthy.

The present talk is concerned with several generalizations of these measures. In the first part we will collect results about classical Henkin measures on strictly pseudoconvex domains, in particular, theorems of Henkin and of Cole and Range. Next, we investigate how the theory of Henkin measures on the polydisc $\mathbb{D}^d$ differs from the classical theory. Finally, we will talk about how to generalize parts of the theory of Mult$(\mathcal{H})$-Henkin measures on $\mathbb{B}_d$ to more general domains and a more general class of reproducing kernel Hilbert spaces.