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Complete Nevanlinna-Pick kernels and the characteristic function

Abstract. We shall extend the classical theory of Sz.-Nagy and Foias about the characteristic function of a contraction to a commuting tuple $(T_1, \ldots, T_d)$ of bounded operators satisfying the natural positivity condition of $1/k$-contractivity for an irreducible unitarily invariant complete Nevanlinna-Pick kernel. The characteristic function is a multiplier from $H_k \otimes \mathcal{E}$ to $H_k \otimes \mathcal{F}$, *factoring* a certain positive operator, for suitable Hilbert spaces $\mathcal{E}$ and $\mathcal{F}$ depending on the tuple $(T_1, \ldots, T_d)$. Surprisingly, there is a converse, which roughly says that if a kernel k *admits* a characteristic function, then it has to be an irreducible unitarily invariant complete Nevanlinna-Pick kernel. The characterization explains, among other things, why in the literature an analogue of the characteristic function for a Bergman contraction ($1/k$-contraction where k is the Bergman kernel), when viewed as a multiplier between two vector valued reproducing kernel Hilbert spaces, requires a different (vector valued) reproducing kernel Hilbert space as the domain.

So, what can be said if $(T_1, \ldots, T_d)$ is $1/k$-contractive when k is an irreducible unitarily invariant kernel, but does not have the complete Nevanlinna-Pick property? We shall see that if k has a complete Nevanlinna-Pick factor s, then much can be retrieved just by allowing the characteristic function to be a multiplier now from $H_s \otimes \mathcal{E}$ to $H_k \otimes \mathcal{F}$, for suitable $\mathcal{E}$ and $\mathcal{F}$ depending on $(T_1, \ldots, T_d)$. This is a joint work with Tirthankar Bhattacharyya.