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Talk Title:

Injectivity, amenability, and trace class operators over locally compact quantum groups

Abstract:

I shall give a survey of our results on Banach algebra structures on the space of trace class operators $\mathcal{T}(L_2(\mathbb{G}))$ over a locally compact quantum group \mathbb{G} . Indeed, the latter carries two dual products which, in the group case, are operator analogues of the convolution product in the group algebra and the pointwise product in the Fourier algebra, respectively. We study in particular the corresponding module structures on $\mathcal{B}(L_2(\mathbb{G}))$. As an application, we characterize quantum group amenability in terms of injectivity in the category of $\mathcal{T}(L_2(\mathbb{G}))$ modules. We also present the very recent result that the space of trace class operators of trace 0 carries naturally a Jordan-type product, stemming from the two dual products, which is associative: this gives rise to a new Banach algebra capturing quantum group duality. We show, for instance, that this algebra has a bounded approximate identity if and only if \mathbb{G} and its dual $\hat{\mathbb{G}}$ are both co-amenable.

The talk is mainly based on joint work with J. Crann, Z. Hu, M. Kalantar, and Z.-J. Ruan.