Titles and Abstracts for RolandFest65

Octavio Arizmendi

(Finite) free Probability, Derivatives
and Random Polynomials

Steinerberger (2019) discovered a surprising connection between the root distribution of derivatives of random polynomials and free additive convolution powers. This connection was later rigorously established by Hoskins and Kabluchko (2021).

In this talk, we'll see how tools from finite free probability not only lead to remarkably simple and elegant proofs of this result, but also open the door to new results about polynomials and random polynomials. Along the way, we'll draw on ideas from random matrix theory and free probability to guide our understanding.

This talk is based on joint works with A. Campbell, K. Fujie, J. Garza-Vargas, D. Perales, and Y. Ueda.

Marwa Banna

Wasserstein rates of convergence in the
free additive and multiplicative CLTs

I begin by presenting r-Wasserstein bounds, $r \geq 1$, for the operator-valued c-free additive central limit theorem (CLT), extending existing results in the operator-valued free and Boolean cases. I will then present the first quantitative estimates for the rate of convergence in the free multiplicative CLT, in terms of the Kolmogorov and r-Wasserstein distances. While the free additive CLT has been thoroughly studied, including convergence rates, the multiplicative setting remained open in this regard. Based on joint works with N. Gilliers and P-L. Tseng.

Hari Bercovici

Subordination for operator-valued multiplicative convolution

I describe how the twisted multiplicativity of the S-transform for freely independent, operator-valued, random variables (first discovered by Dykema) leads to the correct formulation of subordination for products of such variables. In the case of unitary random variables, the subordination functions always extend to the open unit ball of the algebra of ßcalars". I also describe a different proof for the twisted multiplicativity rule, based on an idea of Haagerup. This is a report on joint work with S.T. Belinschi.

Philippe Biane

FREE CUMULANTS EVERYWHERE

I will survey how free cumulants occur, sometimes unexpectedly, in many problems in Mathematics and Physics.

Mike Brannan

QUANTUM GRAPHS AND A QUANTUM VERSION OF FRUCHT'S THEOREM

Given a finite graph X, we can consider its automorphism group Aut(X), which is a finite group. Conversely, given any finite group G, Frucht's theorem asserts that there is a finite simple graph X such that G = Aut(X). If we replace finite groups by finite quantum groups (=finite dimensional Hopf C*-algebras), it is natural to ask what the right generalization of Frucht's theorem is in this context. In order to get a satisfactory answer, it turns out that one must consider both quantum versions of finite graphs and quantum versions of their automorphism groups. Within this framework, we can prove: Every finite quantum group is the quantum automorphism group of a finite simple quantum graph. This result is based on joint work with Daniel Gromada, Junichiro Matsuda, Adam Skalski, and Mateusz Wasilewski.

Marek Bożejko

CYCLIC LENGTH FUNCTIONS ON THE SYMMETRIC AND THE HYPEROCTAHEDRAL GROUPS WITH APPLICATIONS

We present the extension of our results with M. Guta on central Gaussian processes, related to symmetric groups to central Gaussian processes on hyperoctahedral groups with two parameters. We will follows the paper of Bozejko + Dolega + Ejsmont + Gal, *Journal Funct. Anal.* 284, (2023), 47 pp. Applications to construction of new class of generalized Gaussian operators related to Askey-Wimp-Kerov distributions will be also done.

Guillaume Cébron

GRAPHON-THEORETIC APPROACH TO CENTRAL LIMIT THEOREMS FOR ϵ-INDEPENDENCE

ϵ-independence is a mixture of classical independence and free independence corresponding to graph products of groups. I will describe a central limit theorem for the sum of ϵ-independent random variables, extending both the classical and free CLTs. Central to our approach is the use of graphon limits to characterize the limiting distribution, which depends on the asymptotic structure of the underlying graphs governing ϵ-independence. Work in collaboration with Patrick Oliveira Santos and Pierre Youssef.

Adrián Celestino Rodríguez

A shuffle algebra approach to Boolean cumulants of free random variables

Recently, Ebrahimi-Fard and Patras introduced a Hopf–shuffle algebra structure that provides a unified framework for understanding moment–cumulant relations for free, Boolean, and monotone cumulants. In this short talk, we show how the shuffle algebraic perspective allows us to recover a recursive version of a combinatorial formula by Février, Mastnak, Nica, and Szpojankowski, which expresses Boolean cumulants of free random variables.

Ian Charlesworth

Atoms in ε-free probability

ε-free independence is a relation on tuples of algebras indexed by the vertices of a graph which interpolates between classical and free independence: subalgebras should be classically independent if their corresponding vertices form a clique, freely independent if there are no edges between them, and satisfy more complicated moment conditions otherwise. (The same concept has also been variously called Λ-free independence, a graph product, or heap freeness.) This leads to the natural question: can one describe the central summands of a graph product of von Neumann algebras, similar to results in the free setting of Ueda and of Dykema? The answer is yes, at least for type I summands; they correspond to direct summands in the input algebras, and they occur precisely when a finite, explicit list of polynomial inequalities are satisfied by the weights of those summands. I will sketch the argument in the simple case where every input algebra is two-dimensional, which maintains enough complexity to give the flavour of the full argument.

This is joint work with David Jekel.

Benoît Collins

Around the norm of the sum of epsilon-free elements and their matrix models

We consider a natural problem in tensor random matrix theory, that consists in trying to understand the sum of independent GUE with a tensor structure. We give new estimates for their norms and that of their limiting objects in terms of epsilon freeness. Part of this work is joint with Wangjun Yuan, and part is joint with Akihiro Miyagawa. Time allowing, after reviewing motivations and describing our results, we will mention recent related works by other authors in this direction.

Kurusch Ebrahimi-Fard

The operator-valued S-Transform from a group theoretic perspective

We revisit Voiculescu's S-transform in operator-valued free probability using group-theoretic tools. This permits a formulation of its twisted factorization property via crossed morphisms.

Amaury Freslon

The intermediate quantum permutation problem

The intermediate quantum permutation problem asks whether there exists a compact quantum group that lies strictly between the classical permutation group and the free quantum permutation group. After clarifying the meaning of this question, I will explain how Roland's work on free de Finetti theorems and universal notions of independence provides strong evidence against the existence of such an intermediate quantum group. I will then present an unsuccessful attempt by Roland and myself, and explain why it might still be of interest in solving the problem.

Malte Gerhold

Classification of multi-faced independences

In 1997, Roland proved that tensor independence and freeness are the only non-commutative independences which come from a unital and associative universal product on the category of unital algebras. We will discuss related questions for multi-faced universal products and their independences, in particular bifreeness. Based on joint work with Philipp Varšo, Takahiro Hasebe, and Michaël Ulrich.

Takahiro Hasebe

S-transform of probability measures on the real line

Free multiplicative convolution is a fundamental operation in free probability. To compute free multiplicative convolution, S-transform has been a useful machinery. The S-transform was first defined by Voiculescu for compactly supported measures with nonzero mean, and then by Bercovici and Voiculescu for measures on the positive real line, by Raj Rao and Speicher for all compactly supported measures, and by Arizmendi and Perez-Abreu for symmetric measures. After all these contributions, an S-transform of an arbitrary measure was still missing.

In my talk, I will give a definition of the S-transform of an arbitrary measure on the real line, and present its properties. Subordination functions are an important tool to study our S-transform. The talk is based on a joint work with Octavio Arizmendi and Yu Kitagawa.

Claus Köstler

JONES-TEMPERLEY-LIEB ALGEBRAS FROM THE VIEWPOINT OF NONCOMMUTATIVE PROBABILITY

Thoma's theorem provides a characterization of the extremal characters of the infinite symmetric group. Using distributional invariance principles in noncommutative probability, this famous theorem was shown by Gohm and Köstler to form part of a quantum de Finetti theorem. A key element of the underlying operator algebraic approach was the consideration of the infinite symmetric group presented via star generators. Furthermore, Nica and Köstler identified that the central limit laws of this sequence of star generators, with respect to certain extremal characters, correspond to the empirical law of traceless GUE random $d \times d$ matrices. My talk will briefly review these results and address ongoing research into transferring them to extremal tracial states on the Jones-Temperley-Lieb algebras. A successful transfer would include high-lightening the rigidity of the Jones index as a consequence of a noncommutative de Finetti theorem.

References: [1] Rolf Gohm, Claus Köstler. Noncommutative Independence from Characters of the Infinite Symmetric Group S_∞. arXiv:1005.5726. [2] Claus Köstler, Alexandru Nica (2021). A central limit theorem for star-generators of S_∞, which relates to the law of a GUE matrix. *Journal Of Theoretical Probability*, **34** (3):1248-1278

Franz Lehner

CONDITIONALLY FREE CONDITIONAL EXPECTATIONS AND DISTRIBUTIONS

Shortly after the well known free cumulants Roland also discovered conditionally free cumulants (in joint work with Marek Bozejko and Michael Leinert [1]). In joint work with Adrian Celestino and Kamil Szpojankowski [3] we extend our previous work [2] to this setting and compute conditional expectations and distributions of polynomials in conditionally free random variables.

Literatur

[1] M. Bożejko, M. Leinert, and R. Speicher. Convolution and limit theorems for conditionally free random variables. *Pacific J. Math.*, 175(2):357–388, 1996.

[2] F. Lehner and K. Szpojankowski. Free integral calculus I. Free conditional expectations, arXiv:2311.04039.

[3] A. Celestino, F. Lehner and K. Szpojankowski. Free integral calculus II. Conditionally free conditional expectations, arXiv:2311.04039.

Luca Lionni

Tensorial probability spaces

I will introduce the algebraic structures defined in our recent work with Collins and Gurau to describe the first order infinite size limit of random tensors, which generalize non-commutative probability spaces, and then provide the moments formulation of tensor freeness in these spaces.

Jamie Mingo

Real Infinitesimal Freeness

We introduce a new kind of free independence, called real infinitesimal freeness. We show that independent orthogonally invariant with infinitesimal laws are asymptotically real infinitesimally free. We introduce new cumulants, called real infinitesimal cumulants and show that real infinitesimal freeness is equivalent to vanishing of mixed cumulants. We prove the formula for cumulants with products as entries. This joint work with Guillaume Cébron.

Raj Rao Nakadutti

The joys of seeing free probability "work" in the real world

I was introduced to free probability via Roland Speicher's expository writing on it. It was a delight then, as it is now, to see all the various branches of mathematics it elegantly connected with, including in particular random matrices.

Random matrices, in practice, are not as random and not as isotopic as the theory seemingly requires and yet somehow free probability "works" as a predictive tool.

We describe one of many such joys of using the theory to predict experimental results by focusing on an application involving random matrices, the S-transform and how free probability helps develop a mathematical framework for reasoning about when, how and to what extent it is possible to see through a seemingly opaque medium, such as eggshells and yogurt, as though they were transparent.

Alexandru Nica

Overlap measures and free denoising

The term *free denoising* appears in connection to conditional expectations of the form $E(a|a+b)$, with a, b selfadjoint random variables that are freely independent, and where we think of a as of some kind of a "signal", while b is a "noise" which corrupts the signal. In joint work with Maxime Fevrier and Kamil Szpojankowski (arXiv:2412.20792) we show how free denoising can be studied by using a two-dimensional measure associated to a and $a+b$, which we call *overlap measure*. The special case when the noise b is a semicircular element gives the free analogue of a well-known formula of Tweedie from statistics that had also been observed in random matrix literature studying the related notion of matrix denoising.

Jon Novak

From Gauss to Speicher: hypergeometric kernels and free cumulants

Hypergeometric kernels on the space of complex matrices were introduced in multivariate statistics by Alan James in the 1950s. One of the most fascinating open problems in asymptotic analysis is that of finding an effective approximation scheme for these multivariate special functions in the high-dimensional limit. Over the course of the past decade, a (still conjectural) solution has gradually emerged: every hypergeometric kernel admits a topological expansion in terms of certain generalized Hurwitz numbers. A remarkable fact is that in a particular scaling the leading term in the topological expansion of the exponential kernel becomes the R-transform, and the leading term in the topological expansion of the Bessel kernel becomes the rectangular R-transform. In fact, every hypergeometric matrix kernel gives its own R-transform in the high-dimensional limit, but whether or not these special functions have a canonical meaning in free probability remains to be determined. Based in part on joint work with Colin McSwiggen.

Sang-Jun Park

Tensor free independence and central limit theorem

Voiculescu's notion of asymptotic free independence applies to a wide range of random matrices, including those that are independent and unitarily invariant. In the joint work with Ion Nechita, we generalize this notion by considering random matrices with a tensor product structure that are invariant under the action of local unitary matrices. Assuming the existence of the "tensor distribution" limit described by tuples of permutations, we show that an independent family of local unitary invariant random matrices satisfies asymptotically a novel form of freeness, which we term "tensor freeness". Furthermore, we propose a tensor free version of the central limit theorem, which extends and recovers several previous results for tensor products of free variables.

Felix Parraud

On the use of free stochastic calculus in Random Matrix Theory

The theory of free stochastic calculus was developed by Biane and Speicher in the late 90s. In the last few years it has turned out to be a very useful tool to study the asymptotic behaviour of random matrices and compute matrix integrals. In this talk I will explain the heuristic behind those results, as well as list a few of them.

In particular I will explain how I used it to deduce a proof of a conjecture on the strong convergence of a family of random matrices which implies the Peterson-Thom conjecture thanks to the work of Ben Hayes.

Michael Skeide

Freeness in a Conditional Expectation: Quantum Stochastic Calculus and Free Product Systems

Freeness is, in a sense, the most noncommutative notion of *quantum stochastic independence* in a *state*. Even more noncommutative is the situation when also the scalars are allowed to form a not necessarily commutative algebra. Independence becomes, thus, *conditional independence* in a *conditional expectation*. The most classical notion of scalar noncommutative independence, *tensor independence*, where independent subalgebras (possibly noncommutative) do commute with each other, even goes away by passing to noncommutative scalars: There is no conditional tensor independence in general and conditional freeness is the only noncommutative independence in which the subalgebras sit unitally in the big algebra. (For *monotone* and *boolean* independence – scalar or conditional – at least some of the subalgebras, though still unital in their own right, sit nonunitally in the big one.)

In this talk, we explore some special aspects of conditional freeness and its relation with conditional monotone independence that occurred in our research and were – some more, some less – inspired by some of Roland's works and in any case where favoured by the productive atmosphere in the "Heidelberg-Group" around Wilhelm von Waldenfels to which also Roland contributed a lot.

Piotr Śniady

Postcards from asymptotic representation theory: a curated tour of mathematical landmarks

These vintage-style postcards showcase montages of celebrated landmarks, strategically composed to convey the cultured lifestyle and cosmopolitan experiences of the traveler fortunate enough to send them. This talk adopts a similar curatorial approach, presenting highlights from asymptotic representation theory specifically for an audience well-versed in free probability theory. The landmarks we visit—free cumulants, non-crossing partitions, graphs embedded on surfaces, and various asymptotic and probabilistic questions—are not exotic destinations but familiar mathematical territory. Like receiving a postcard from your own hometown taken by a visiting photographer, you may discover fresh perspectives on mathematical landscapes you thought you knew completely.

Dan Voiculescu

Remarks about topological free entropy

Topological free entropy is a version of the microstates free entropy for C*-algebras, based on "norm-microstates". I will discuss some of the results and open problems.

Jiun-Chau Wang

The impact of the paper "The normal distribution is freely infinitely divisible"

The paper mentioned in the title came out as a surprise to many in 2011. It has inspired a pursuit of free infinite divisibility over the years. Here we shall report its latest application to free convolution with the normal distribution. This is a joint work with Takahiro Hasebe.

Moritz Weber

Partitions go quantum

In 2009, Teo Banica and Roland Speicher defined the so called „easy“ quantum groups. They are based on exactly the combinatorics for which Roland is well-known: partitions of sets. The introduction of „easy“ quantum groups has initiated a whole wave of research which I will survey briefly, including some very recent work, for instance on spatial partitions. I will also mention my 2016 article with Roland on Quantum Groups with Partial Commutation Relations, which is linked with quantum automorphism groups of graphs. I will briefly survey this direction, too, mentioning some recent related work on hypergraphs and hypergraph C*-algebras.